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SCHEDULING STOCK ISSUES IN THE PRESENCE OF TRANSACTION COSTS

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Introduction

Most of the writing in Finance Theory treat transaction costs qualitatively. Quantitative models traditionally ignore the transactions costs in order to realize compact presentation. While in most of the cases such an approach is justified due to the triviality of these costs, the costs involved with stock issues are an exception.

In the context of stock issues, assuming away the transaction costs implies that if a firm issues in some period, it will issue the exact amount needed to satisfy the cash demand for that period. No transaction costs means no motivation for creating cash inventories by issuing.[1] While qualitative arguments can explain the fact that firms issue for inventory, they cannot tell managers how much and when to issue. Further, they cannot cope with related problems, for example: if stock holders do not incur any cost due to holding liquid inventory does it mean that the firm should issue once a huge amount of shares to satisfy the expected cash needs over its expected life? How much forecasted data about future cash demands and market conditions is necessary in order to make an optimal issue today? Does an unrelated merger reduce the need for forecasts? Does a change in taxation influence the frequency of issuing?

Some writers do treat transaction costs in the context of optimization but the emphasis is different than this article's (For example: Smith [8] discusses the best method issuing stock by comparing costs of underwritten versus rights issues, Ibbotson and Jaffe [4] discuss the question of

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[1] This is true only for efficient markets, since differential information may motivate issuing for direct profit making (vs. satisfying cash demands).

"hot issue" markets - periods in which the performance of a new issue is abnormally high.) My purpose is rather different. I am concerned with costs of processing the issues. These costs include underwriting fees, legal fees, accounting fees, trustee's fees, listing fees, printing and engraving expenses, SEC registration fees, federal stamps, taxes and employee's time loss and underpricing. These costs are significant. Smith [8] found, based on a sample of 578 stock issued during 1971-1975 that for underwritten issues these costs (not including employees' time loss and underpricing) as a percentage of the proceeds to range from around 14% to around 3.5%, depending on the size of the issue, with an average of 6.17%. The average for rights issues was 2.45% (without standby agreement). However, the fact is that only 5% of the issues are by the rights method.

These costs contain a fixed cost and variable cost and can be closely modeled as a convex function of the size of the transaction, with time dependent parameters. Underwriting fees (which form more than 80% of the 6.17% average mentioned above) can also be expressed as fixed and variable cost. The fixed and variable parameters in each period will depend on the demand function for equity in each period and the cost to the investment bank. Their determination is a problem of two part tariffs (for discussion of this problem see [6]) and will not be discussed here . The point to be made is that their changes over time are not just a matter of adjustment for inflation. The stochastic process describing the behavior of these cost components is complex and not yet understood, however, it seems plausible to assume that the covariance with the market return is zero. This assumption will be used throughout the rest of this article, where future transaction costs are discounted at the risk free rate.

In the following sections the stock issuance scheduling problem is modeled and solved for the deterministic case and a simple algorithm is

suggested. This algorithm is shown to be a search for forecasting and planning horizons. When we say that T is a forecasting horizon and $t \leq T$ is a planning horizon, we mean that data for periods $T+1$, $T+2$ ... is irrelevant for making optimal decisions in periods 1 through t. The implication of the existance of such forecasting horizons is discussed and the effects of mergers on the scheduling are analyzed. It will be shown that conglomerate mergers may create synergy that goes beyond the savings on the fixed component of the transaction costs. Although the value of this synergy is relatively low, when added to other marginal synergies (slack creation for example) may strengthen the rationale for non-related mergers. Next the stochastic case is treated. The difficulties involved in obtaining an analytical solution are demonstrated and a heuristic technique based on the deterministic model is suggested.

Formulation and Notation

Consider a firm which decides to undertake growth opportunities. In order to realize it investments have to be made from now through period N (for general discussion N can be taken to infinity). These investments will be financed from internal resources, debt and equity. The residual cash needs to be financed by stock issues are assumed to be forecastable with certainty as well as the term structure and the tax rate (for simplicity the tax rate will be treated as constant but this can be easily relaxed). The transaction costs involved in processing an issue are assumed to be a convex function of the transaction value. Cash inventories are invested in risk free assets. The objective of the firm is assumed to be maximization of the existing stock-holders equity. Market efficiency and completeness are assumed throughout.

The following notation will be used.

(1)	$\{d_t \; ; \; t = 1, 2, \ldots N\}$	- cash demands in period t <u>to be satisified on time</u> by stock issues
(2)	$\{k_t \; ; \; t = 1, 2, \ldots N\}$	- fixed transaction costs
(3)	$\{\alpha_t \; ; \; t = 1, 2, \ldots N\}$	- variable transaction costs per $1 issued
(4)	$\{r_t \; ; \; t = 1, 2, \ldots N\}$	- risk free discount rates
(5)	$\{I_t \; ; \; t = 1, 2, \ldots N\}$	- liquid assets inventory entering period t (only the inventory created by stock issues)
(6)	T_c	- corporate tax rate, assumed constant
(7)	$\{Y_t \; ; \; X_t \; ; \; t = 1, 2, \ldots N\}$	- net cash inflow and total value of equity issued, respectively
(8)	$\gamma_t = \prod_{j=1}^{t-1} (1 + r_j), \; \gamma_1 = 1$	- discount factor for period t
(9)	$PV_t \{\cdot\}$	- present value as of period t of a stream $\{\cdot\}$
(10)	$\{E_t, \hat{E}_t \; ; \; t = 1, 2, \ldots N\}$	- total market value of equity, and value of equity belonging to existing stockholders, in period t, respectively.

The following lemma will form the basis for the problem formulation:

<u>Lemma 1</u>: In a complete and efficient capital market, if $\frac{\partial Y_t}{\partial X_t} < 1 \forall X_t > 0$ and $\forall \; t \varepsilon [1,N]$ then ceteris paribus, maximization of existing equity in each period is equivalent to:

$$\min PV_1 \{X_t | t = 1, 2, \ldots N\}$$

Proof: Acting on behalf of existing equity in each period, in an efficient complete market implies maximization of the stock price in period 1. Further the efficiency and completeness assumptions allows for risk independence implying $PV_t(A \pm B) = PV_t(A) \pm PV_t(B)\ \forall_t$ (for further discussion of this points see [3] and [5]) allowing for the following derivation.

Now, the value in period N of the existing equity in period N-1 satisfies: $\hat{E}_N = E_N - X_N$

Similarily: $\hat{E}_{N-1} = PV_{N-1}(E_N) - X_{N-1}$

$$\hat{E}_{N-2} = PV_{N-2}(E_{N-1}) - X_{N-2}$$

$$\vdots$$

$$\hat{E}_1 = PV_1(E_2) - X_1$$

Equivalently: $\hat{E}_1 = PV_1(E_N) - PV\ \{X_t\ | t = 1, 2, \ldots N\}$

$$\frac{\partial \hat{E}_1}{\partial PV_1\{X_t | t=1,2,\ldots N\}} = \frac{\partial PV_1(E_N)}{\partial PV_1\{Y_t | t=1,2,\ldots N\}} \quad \frac{\partial PV_1\{Y_t | t=1,2,\ldots N\}}{\partial PV_1\{X_t | t=1,2,\ldots N\}} - 1$$

$$= \frac{\partial PV_1\{Y_t | t=1,2,\ldots N\}}{\partial PV_1\ \{X_t | t=1,2,\ldots N\}} - 1$$

and due to the condition $\frac{\partial Y_t}{\partial X_t} < 1\ \forall\ Y_t\ \&\ t\varepsilon[1,N]$ implies $\frac{\partial \hat{E}_1}{\partial PV\{X_t | t=1,2,\ldots N\}} < 0$. Hence $\hat{E}_1$ is a decreasing function of $PV(X_t | t = 1,2,\ldots N)$ implying the required result. Q.E.D.

Since the transaction costs are modeled as $C_t = K_t + \alpha_t X_t,\ \forall\ X \geq 0$ to obtain a net inflow y_t the firm will issue: $X_t = \frac{Y_t + K_t}{1 - \alpha_t}$. Note also that $\frac{\partial Y_t}{\partial X_t} = 1 - \alpha_t < 1\ \forall\ Y_t\ t\varepsilon[1,N]$ satisfying the Lemma's conditions. Hence the problem can be formulated as:

$$\text{(P1)} \quad \text{Min} \quad \sum_{t=1}^{N} \frac{X_t\delta_t}{Y_t}$$

$$\text{S.T.} \quad X_t = \frac{y_t + k_t}{1-\alpha_t} \qquad t = 1, 2, \ldots N$$

$$\delta_t = \begin{cases} 1 \text{ if } y_t > 0 \\ 0 \text{ if } y_t = 0 \end{cases} \qquad t = 1, 2, \ldots N$$

$$I_t + Y_t \geq d_t \qquad t = 1, 2, \ldots N$$

$$I_{t+1} = (I_t + Y_t - d_t)[1+r_t(1-T_c)] \qquad t = 1, 2, \ldots N$$

$$I_1 = 0$$

$$Y_t \geq 0 \qquad t = 1, 2, \ldots N$$

or

$$\text{(P2)} \quad \text{Min} \quad \sum_{t=1}^{N} \left[\frac{K_t\delta_t}{(1-\alpha_t)\gamma_t} + \frac{Y_t}{(1-\alpha_t)\gamma_t}\right]$$

$$\text{S.T.} \quad \delta_t = \begin{cases} 1 \text{ if } Y_t > 0 \\ 0 \text{ if } Y_t = 0 \end{cases} \qquad t = 1,2,\ldots N$$

$$I_t + Y_t \geq d_t \qquad t = 1,2,\ldots N$$

$$I_{t+1} = (I_t + Y_t - d_t)[1+r_t(1-T_c)] \qquad t = 1,2,\ldots N$$

$$I_1 = 0$$

$$Y_t \geq 0 \qquad t = 1,2,\ldots N$$

(Due to the first constraint in P2, Y_t in the objective does not have to be multiplied by δ_t.)

Extantion of the E.G.P. Planning Horizon Theorem

Eppen, Gould and Pashigian [2], have stated conditions under which planning horizon will exist for an aggregate production planning problem with time dependent set-up and production costs. (P2) is different in two ways:

(1) There are no costs for holding inventories[1]

(2) Inventories are appreciating with time.

Using a similar method of proof as E.P.G. conditions for planning horizons are defined and an efficient algorithm for solving the problem is developed.

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[1] In fact the stockholders are penalized for holding inventory due to taxation, however, this penalty is incorporated in the problem since it forces issuing more equity.

Notation

(i) $F(\ell,t)$: the minimum cost in a t period problem, with a final issue in period ℓ.

(ii) $F(t)$: the minimum cost for a t period problem.

(iii) $\ell(t)$: the period in which final issuance takes place in an optimal solution to the t period problem.

it follows from the definition that:

(1) $F(t) = F(\ell(t), t) = \min_{\ell} F(\ell, t) \quad \ell \in 1, 2, \ldots N$

(2) $F(\ell, t) = \dfrac{X_\ell}{\gamma_\ell} + F(\ell-1)$

Now suppose $\ell < t_1 < t_2$, from (2) it follows that:

(3) $F(\ell, t_2) = F(\ell, t_1) + \dfrac{Y(t_1+1, t_2)}{\gamma_\ell (1-\alpha_\ell)}$

where $Y(t_1 + 1, t_2)$ is the additional net amount to be raised in period ℓ as to satisfy the demand from $t_1 + 1$ through t_2. Note that:

$$Y(t_1+1, t_2) = \frac{Y_{t_1+1}}{\prod_{j=\ell}^{t_1} (1+r_j(1-T_c))}$$

where $\hat{Y}_{t_1+1}$ is the amount that should have been raised if we issued again at t_1+1.

Hence:

(3) $$F(\ell, t_2) = F(\ell, t_1) + \frac{\hat{Y}_{t_{1+1}}}{\prod_{j=\ell}^{t_1} (1+r_j(1-T_c))} \cdot \frac{1}{\gamma_\ell(1-\alpha_\ell)}$$

Now Let:

$$a_{j,t} = \prod_{i=j}^{t-1} [1+r_i(1-T_c)]\gamma_j (1-\alpha_j) \ ; \ a_{t,t} = \gamma_t(1-\alpha_t)$$

we say that condition Ψ holds if $\quad a_{j,\ell(t_1)} \leq (1-\alpha_{\ell(t_1)})\gamma_{\ell(t_1)}$

Lemma 2: Suppose a minimum cost for a t_1 period problem is obtained by a program with a final issue at $\ell(t_1) \le t_1$ and Ψ holds. Then there exist a minimum cost program for a $t_2 > t_1$ problem which has a final issue at $\ell(t_2) \ge \ell(t_1)$.

Proof: Suppose there is a period $j < \ell(t_1)$ such that $F(t_2) = F(j, t_2)$. Since Ψ holds either $a_{j,\ell(t_1)} < \left(1-\alpha_{\ell(t_1)}\right)\gamma_{\ell(t_1)}$ or $a_{j,\ell(t_1)} = \left(1-\alpha_{\ell(t_1)}\right)\gamma_{\ell(t_1)}$.

(i) $a_{j,\ell(t_1)} < \left(1-\alpha_{\ell(t_1)}\right)\gamma_{\ell(t_1)}$. By definition $F(t_1) = F\,[\ell(t_1), t_1]$.

$F(t_2) = F(j, t_2)$ and $t_1 < t_2$ by assumption.

By definition $F(t_2) = F\,(j, t_2) \le F\,[\ell(t_1), t_2]$ and by using EQ.(3);

$$F(j, t_1) + \frac{\hat{\gamma}_{t_1+1}}{a_{j,\, t_1+1}} \le F[\ell(t_1), t_1] + \frac{\hat{\gamma}_{t_1+1}}{a_{\ell(t_1),\, t_1+1}}$$

$$\Rightarrow \quad F(j,t_1) + \hat{\gamma}_{t_1+1}\left[\frac{1}{a_{j,\, t_1+1}} - \frac{1}{a_{\ell(t_1),\, t_1+1}}\right] \le F(\ell(t_1)\,,\, t_1)$$

But $F(j,t_1) + \hat{\gamma}_{t_1+1}\left[\dfrac{1}{a_{j,t_1+1}} - \dfrac{1}{a_{\ell(t_1),\, t_1+1}}\right]$

$$= F(j,t_1) + \hat{\gamma}_{t_1+1}\left[\frac{\prod_{i=\ell(t_1)}^{t_1}(1+r_i(1-Tc))}{\prod_{i=j}^{t_1}(1+r_i(1-Tc))\gamma_j(1-\alpha_j)} - \frac{1}{(1-\alpha_{\ell(t_1)})\gamma_{\ell(t_1)}}\right]\frac{1}{\prod_{i=\ell(t_1)}^{t_1}(1+r_i(1-T_c))}$$

$$= F(j,t_1) + \hat{\gamma}_{t_1+1}\left[\frac{1}{\prod_{i=j}^{\ell(t_1)-1}(1+r_i(1-Tc))\gamma_j(1-\alpha_j)} - \frac{1}{(1-\alpha_{\ell(t_1)})\gamma_{\ell(t_1)}}\right]\frac{1}{\prod_{\ell(t_1)}^{t_1}(1+r_i(1-T_c))}$$

$$= F(j,t_1) + \frac{\hat{Y}_{t_1+1}}{\prod_{i=\ell(t_1)}^{t_1} (1+r_i(1-Tc))} \left\{ [a_{j,\ \ell(t_1)}]^{-1} - \frac{1}{(1-\alpha_{\ell(t_1)})\gamma_{\ell(t_1)}} \right\} > F(j,t_1).$$

In consequence:

$$F(j,t_1) < F(\ell(t_1),t_1)$$

an obvious contradiction.

(ii) $a_{j,\ell(t_1)} = (1-\alpha_{\ell(t_1)})\gamma_{\ell(t_1)}$ in this case:

$$F(j,t_1) = F(\ell(t_1),t_1)$$

meaning that there exist an alternative optimal program for the t_1 period problem with final issue in period $j < \ell(t_1)$

We observe that:

$$a_{j,t_1+1} = a_{j,\ell(t_1)} \prod_{i=\ell(t_1)}^{t_1} [1+r_i(1-Tc)]$$

$$= (1-\alpha_{\ell(t_1)})\gamma_{\ell(t_1)} \prod_{i=\ell(t_1)}^{t_1} (1+r_i(1-Tc)) = a_{\ell(t_1)\ t_1+1}$$

thus using (3),

$$F(j,t_2) = F(j,t_1) + \frac{\hat{Y}_{t_1+1}}{a_{j,t_1+1}} = F[\ell(t_1),t_1] + \frac{\hat{Y}_{t_1+1}}{a_{\ell(t_1),\ t_1+1}}$$

$$= F[\ell(t_1),t_2]$$

$$\Longrightarrow \ell(t_2) = \ell(t_1)$$

Q.E.D.

<u>LEMMA 3</u>: If a minimum cost for a t_1 periods problem is obtained by a program with final issue at $\ell(t_1)=t_1$ and ψ holds. Then there exists a minimum cost program for a $t_2>t_1$ periods problem with an issue (not necessarily final) in period t_1.

PROOF: Consider a t_1+1 period problem. It is either optimal to issue in period t_1+1, for requirements in t_1+1, then the problem reduces to t_1 periods problem, or to issue in t_1 for requirements in t_1 and t_1+1. By lemma 2 we do not have to consider the periods before t_1. By induction for t_1+2, t_1+3... period problems it is easily shown that an issuance will take place in period t_1. Q.E.D.

LEMMA 4: Under the conditions of Lemma 3 there is an optimal policy such that $I_{t_1} Y_{t_1} = 0$. (issue only when inventory is zero)

PROOF: Suppose we have an optimal program with $Y_{t_1} I_{t_1} > 0$ for a t_1 periods problem and the issue before the final took place in period j, then in period j sum $A = I_{t_1} \Big/ \prod_{i=j}^{t-1} (1+r_i(1-Tc))$ was invested in safe liquid asset for t_1 - j periods. Since $Y_j > A$, rescheduling by raising $Y_j - A$ in period j and $Y_{t_1} + I_{t_1}$ in period t_1 we incur the incremental cost:

$$\Delta = -\frac{A}{\gamma_j(1-\alpha_j)} + \frac{I_{t_1}}{(1-\alpha_{t_1})\gamma_{t_1}}$$

$$= I_{t_1}\left(\frac{1}{(1-\alpha_{t_1})\gamma_{t_1}} - \frac{1}{a_{j,t_1}}\right)$$ But since $\ell(t_1) = t_1$:

$$\Delta = I_{t_1}\left(\frac{1}{(1-\alpha_{\ell(t_1)})\gamma_{\ell(t_1)}} - \frac{1}{a_{j,\ell(t_1)}}\right)$$

$\Delta \leq 0$ since Ψ holds.

Hence it is not more costly to reschedule such that

$I_{t_1}Y_{t_1} = 0$

LEMMA 5: Under the conditions of Lemma 3, an optimal program for $t_1 - 1$ periods problem will be part of an optimal program for any longer problem.

PROOF: The proof follows immediately from Lemmas 3 and 4. Q.E.D.

LEMMA 6: Under the conditions of Lemma 3, there is an optimal program such that:

1) $I_t Y_{t'} = 0 \quad t \leq t_1$

2) $Y_t = 0$ or $Y_t = \sum_{j=t+1}^{k-1} \frac{d_j}{\prod_{i=t}^{j-1}[1+r_i(1-Tc)]} + d_t$

where $k \leq t_1$ is the first period after t in which there is issuance.

PROOF: Part 2 follows immediately from part 1. The proof for part 1 follows from Lemma 4 and Lemma 5 by breaking the t_1 periods problem to subproblems with length t_i for each i such that $Y_i > 0$.

Q.E.D.

DISCUSSION

Under the conditions of Lemma 3, it follows that periods 1 through $t_1 - 1$ can be planned optimally without any information about costs, demands or term structure in periods later than $t_1 \leq N$. Periods 1 through $t_1 - 1$ constitute a planning horizon.

The existence of a planning horizons is significant because in most cases managers decide to undertake future opportunities without having a full cash flows forecast. Assessing the need for cash to be raised by stock issues over the life of the firm is an impossible task, however, an optimal issuing program can be obtain in most cases with rather limited information about the future. If a Forecasting Horizon is found, information beyond it is not needed to make an optimal issue in period 1.

The implication is that managers should invest inforecasting only up to period t_1 and make their second forecast for periods beyond t_1 later, thus always making "short horizon" forecasts at a lower cost and increased accuracy.

Economic Interpretation of Condition Ψ

Condition Ψ can be rewritten as:

$$\frac{1}{\gamma_{\ell(t_1)}(1-\alpha_{\ell(t_1)})} \leq \frac{1 \Big/ \prod_{j}^{\ell(t_1)-1} (1+r_i(1-Tc))}{(1-\alpha_j)\,\gamma_j} \qquad j<\ell(t_1)$$

which says: the present value as of period 1 of the equity sold in period $\ell(t_1)$ in order to raise an additional one net dollar is smaller than the present value of the equity sold in period j in order to have a net one dollar in period $\ell(t_1)$.

By transferring the left handside to the right we get an expression for the marginal gains from delaying the cash inflow $\ell(t_1)$-j periods. Condition Ψ is satisfied when this marginal gain is positive.

We observe also that Ψ can be also written as:

$$1-\alpha_{\ell(t_1)} \geq (1-\alpha_j) \prod_{i=j}^{\ell(t_1)-1} \left(\frac{1+r_i\ (1-Tc)}{1+r_i} \right)$$

hence Ψ holds whenever $\alpha_{\ell(t_1)} \leq \alpha_j$.

Clearly, planning horizons are more likely to be found in economies where the variable issuing costs are not increasing rapidly. Corporations who have high reported losses or other tax credits are likely to incur longer planning horizons than corporations who don't. An increase in the interest rates in the economy will increase the likelihood of condition Ψ being satisfied faster, and hence we can expect, ceteris paribus, to see more firms making relatively small but frequent issues. Since inflation increases the nominal interest rates it will also tend to increase the frequency of issuing.

If condition Ψ is violated for some value of $j<\ell(t_1)$, Lemma 2 does not apply, in such a case it is possible to have a $t_2>t_1$ such that for $j<\ell(t_1)$ we have $F(j,t_2)<F(k,t_2)$ $K\geq\ell(t_1)$. We want to narrow down the number of periods $j<\ell(t_1)$ which should be considered as candidates for the last issue.

<u>LEMMA 7</u>: Suppose $t_1<t_2$ and $\ell(t_2)<\ell(t_1)$ then

$$a_{\ell(t_2),\ell(t_1)} > \left(1-\alpha_{\ell(t_1)}\right) \gamma_{\ell(t_1)}$$

<u>PROOF</u>: Trivial by Lemma 2.

<u>LEMMA 8</u>: In a t_1 periods problem, no issuance will take place in period $j<\ell(t_1)$ for which Ψ is violated.

<u>PROOF</u>: If there was an issue in period j, it would not pay to issue again in period $\ell(t_1)$ Q.E.D.

<u>LEMMA 9</u>:

$$\frac{K_{\ell(t_1)}}{1-\alpha_{\ell(t_1)}} < \frac{K_j}{1-\alpha_j} \quad \forall\ j \ \text{s.t}\ \Psi \text{ is violated.}$$

<u>PROOF</u>: follows from Lemma 8. Q.E.D.

Since we always issue in period 1 ($I_1=0$ and $d_1>0$) then if $\ell(t_1)>1$, Ψ cannot be violated, so violation of Ψ can occur only for $1<j<\ell(t_1)$

where

$$\frac{K_j}{1-\alpha_j} > \frac{K_{\ell}(t_1)}{1-\alpha_{\ell}(t_1)}$$

Now we introduce the 'violators' set' V.

DEFINITION: $V\ (t_1,t_2) = \left\{ t \varepsilon \{1,2\ldots t_2\} :\ a_{t\ ,t_2} > a_{t_1,t_2} \right\}$

THEOREM 1:

If for any t, $F(\ell,t+1) < F(\ell(t),t+1)$ then $\ell \varepsilon V(\ell(t), t+1)$. In particular this implies for any periods t_1, t_2 such that $t_1<t_2$, either $F(t_2)=F(\ell(t_1),t_2)$ or $\ell(t_2) \varepsilon V(\ell(t_1),t_2)$

PROOF:

i) Suppose $\ell \neq t+1$ and $\ell \neq \ell(t)$.

By expression 3 we have

$$F(\ell,t+1) = F(\ell,t) + \frac{\hat{Y}_{t+1}}{a_{\ell,t+1}}$$

$$F(\ell(t),t+1) = F(\ell(t),t) + \frac{\hat{Y}_{t+1}}{a_{\ell(t),t+1}}$$

since by definition $F(\ell,t) \geq F(\ell(t),t)$ and $F(\ell,t+1) < F(\ell(t),t+1)$ from the theorem's condition, it follows that $a_{\ell,t+1} > a_{\ell(t),t+1} \Rightarrow$

$\ell \varepsilon V\ (\ell(t),t_2)$

ii) Suppose $\ell = t+1$

$$F(t+1,t+1) = F(t) + \frac{d_{t+1} + K_{t+1}}{(1-\alpha_{t+1})Y_{t+1}}$$

$$F[\ell(t),t+1] = F(\ell(t),t) + \frac{d_{t+1}}{a_{\ell(t),t+1}}$$

Since $F(t) = F(\ell(t),t)$ by definition and $F(t+1,t+1) < F(\ell(t),t+1)$ from the theorem's condition, it must be that

$$\frac{d_{t+1} + k_{t+1}}{\gamma_{t+1}(1-\alpha_{t+1})} \leq \frac{d_{t+1}}{a_{\ell(t),t+1}}$$

$$\text{or } \gamma_{t+1}\left(1-\alpha_{t+1}\right) > a_{\ell(t),t+1}$$

$$\Rightarrow a_{t+1,t+1} > a_{\ell(t),t+1} \quad \text{and} \quad \ell \varepsilon V(\ell(t),t+1)$$

The second assertion is proved by induction. It is proved to be true for $t_2 = t_1+1$, assume it is true for $t_2 = t_1+k$, and consider the t_1+k+1 problem. We must show that either $\ell(t_1+K+1) = \ell(t_1)$ or $\ell(t_1+K+1) \varepsilon V(\ell(t_1), t_1+K+1)$. By the first part of the theorem there are two possibilities:

i) $\ell(t_1+K+1) = \ell(t_1+K)$. In this case either $\ell(t_1+K+1) = \ell(t_1+K) = \ell(t_1)$ or $\ell(t_1+K+1) = \ell(t_1+K) \varepsilon V\ (\ell(t_1),t_1+K)$

$$\Rightarrow a_{\ell(t_1+K+1),t_1+K+1} = a_{\ell(t_1+K),t_1+K+1} =$$

$$= [a_{\ell(t_1+K),t_1+K}][1 + r_{t_1+K}(1-T_c)] > a_{\ell(t_1),t_1+K}][1 + r_{t_1+K}(1-T_c)]$$

$$= a_{\ell(t_1),t_1+K+1} \Rightarrow \ell_{(t_1+K+1)} \varepsilon V(\ell(t_1),t_1+K+1)$$

In either case the theorem is true.

ii) $\ell(t_1+K+1) \varepsilon V(\ell(t_1+K), t_1+K+1)$. By the induction hypothesis either $\ell(t_1+K) = \ell(t_1)$ or $\ell(t_1+K) \varepsilon V(\ell(t_1), t_1+K)$. In this case it is immediate that $\ell(t_1+K) \varepsilon V\ (\ell(t_1), t_1+K+1)$ but this implies that $\ell(t_1+K+1) \varepsilon V\ (\ell(t_1),t_1+K+1)$.

Q.E.D.

<u>Corollary 1</u>

If $t_o\ \varepsilon V\ (\ell(t), t+1)$ then $t_o\ \varepsilon V(\ell(j),t+1)$ for $j = 1,2..,t-1$.

PROOF:

It is sufficient to show that $a_{\ell(j),t+1} \leq a_{\ell(t),t+1}$.

Suppose $a_{\ell(j),t+1} > a_{\ell(t),t+1}$, then $\ell(j) \in V(\ell(t),t+1)$ which implies by the theorem that $F[\ell(j),t+1] < F[\ell(t),t+1]$

$$F[\ell(j),t+1] = F[\ell(j),t] + \frac{\hat{Y}_{t+1}}{a_{\ell(j),t+1}} < F[\ell(t),t] + \frac{\hat{Y}_{t+1}}{a_{\ell(t)}} = F[\ell(t),t+1]$$

$F[\ell(j),t] < F[\ell(t),t]$. Contradiction.

Q.E.D.

THE GENERAL PLANNING HORIZON THEOREM

Notation: Let m (t) be a value of $j \in \{1,2,\ldots t\}$ which maximizes $a_{j,t}$. m (t) will be referred to as the strong maximum for the t period problem.

Let P(t) be the smallest P>t for which $P \in V(m(t),P)$.

THEOREM 2

If $F(t_1) = F(m(t_1),t_1)$ then in any longer problem $(t_2>t_1)$ in which an issuance (not necessarily final) will take place in period $m(t_1)$. Hence, periods 1 through $m(t_1)-1$ constitute a planning horizon, (Periods 1 through t_1 constitute a forecasting horizon).

PROOF

If $t_1<t_2<P(t_1)$ then by the definition of $P(t_1)$ we know that $V(m(t_1),t_2)$ is empty, and thus, from Theorem 1 we see that $F(t_2) = F(m(t_1),t_2)$.

If $t_2 = P(t_1)$ then $V(m(t_1),P(t_1))$ contains only the point $P(t_1)$.

Hence, either $F(t_2) = F(m(t_1), t_2)$ or $\ell(t_2) = t_2 = P(t_1)$. In the first case the theorem is satisfied if we issue for the last time in $P(t_1)$ we are left with a $P(t_1)-1$ period problem which has an issue in $m(t_1)$ by the first paragraph of the proof. Now consider a $P(t_1)+1$ period problem. If $P(t_1)+1 \notin V(m(t_1), P(t_1)+1)$, then by theorem 1 $\ell(P(t_1)+1) \neq P(t_1)+1$. Hence the last issue is in $P(t_1)$ or $m(t_1)$ and as in the above paragrpah, the theorem holds. If, however $P(t_1)+1 \varepsilon V(m(t_1), P(t_1)+1)$ then the last issue may take place in $m(t_1)$, $P(t_1)$ or $P(t_1)+1$ and the total cost should be compared for proper selection. However we note that each possibility involves an issue in period $m(t_1)$, since if we issue in $P(t_1)$ or $P(t_1)+1$ we are left with $P(t_1)-1$ or $P(t_1)$ period problem, for which we have shown the existance of issuance in $m(t_1)$. By extending the argument to $P(t_1)+k$ $k=2,3,\ldots$ the theorem is proved Q.E.D.

We observe that Lemma 3 is a special case of theorem 2. The Lemma's requirement for planning horizon are $\ell(t_1) = t_1$ and Ψ holds, but this implies that $m(t_1)$ can be taken as t_1 so that theorem 2 is satisfied. In general theorem 2 is more powerful than Lemma 3, it can discover planning horizons that cannot be discovered by the Lemma. Such planning horizons have the property that periods i through $m(t_1)-1$ can be planned optimally with information on periods 1 through t_1. Contrary to the Lemma, Theorem 2 allows for $t_1 > m(t_1)$. The greater power of the theorem is at the cost of more information but not necessarily all the information on periods 1 through N.

A Forward Algorithm

The algorithm makes use of theorem 1 to reduce computations. Theorem 1 says that given an optimal program for t periods then for $t+1$ period problem there is an optimal program with last issuance in $\ell(t)$ or in a period belonging to the violators' set. By using collary 1 we can restrict

our attention to the violators' set $V(\ell(t), t+1)$. We use theorem 2 to identify planning Horizon. According to the theorem, if $\ell(t) = m(t)$ then $\ell(t)-1$ is a planning horizon and t is a forecasting horizon (obviously if $\ell(t)=t$ $\ell(t)-1$ is a planning horizon).

The Algorithm

1. $t = 1$
2. List the periods $j \in V(\ell(t-1),t) \cup \ell(t-1)$
3. Record $m(t)$
4. Find $F(t) = \min_j F(j, t)$ $j \in V(\ell(t-1), t) \cup \ell(t-1)$ and call the minimizing $j : \ell(t)$.
5. Record $F(t)$, $\ell(t)$
6. $t = t+1$ and return to step 2.
7. Stop when $m(t)=\ell(t)$, or continue till you run out of data

Remark

(1) In step 4, $F(j,t)$ can be calculated as: $F(j,t)=F(j-1)+ \dfrac{k_j + d_j}{a_{j,j}} + \sum_{i=j+1}^{t} \dfrac{d_i}{a_{j,i}}$

(2) Issuing periods are traced by $\ell(t)$.

Numerical Examples

Two examples are provided. The firms in these examples are assumed to have forecasted data for six future periods. In the first example it is shown that this data is not enough to make an optimal decision in period 1. In the second example it is shown that this data is too much; an optimal decision in period 1 can be made with less forecasting effort. The relationship between these two examples is discussed in the next section.

The cost data is based on Smith's paper [8].

Data for the two examples: (demand and fixed transaction costs are given in million dollar units)

t	1	2	3	4	5	6
k_t	0.2	0.3	0.25	0.40	0.30	0.20
α_t	0.025	0.030	0.040	0.035	0.030	0.030
d_t(example I)	3	5	3	1	1	1
d_t(example II)	6	10	6	2	2	2
r_t	0.06	0.08	0.09	0.11	0.08	0.09
γ_t	1.0000	1.0800	1.1772	1.3067	1.4112	1.5382
$1+r_t(1-T_c)$	1.030	1.040	1.045	1.055	1.040	1.045

The $a_{j,t}$ table*

j \ t	1	2	3	4	5	6
1	0.97500	1.00425	1.04442	1.09142	1.15145	1.19750
2		0.97956	1.01874	1.06458	1.12314	1.16806
3			1.13011	1.18096	1.24592	1.29575
4				1.26097	1.33032	1.38354
5					1.36886	1.42361
6						1.49205
m(t)	1	1	3	4	5	6

$$*a_{j,t} = \prod_{i=j}^{t-1} [i+r_i(1-T_c)]\gamma_j(1-\alpha_j) \qquad t>j$$

$$a_{j,j} = \gamma_j\,(1-\alpha_j)$$

I. Algorithm Execution, Example I

j \ t	1	2	3	4	5	6
1	$\frac{0.2+3}{.975}$ =3.282051	3.280251+ $\frac{5}{1.00425}$ = 8.260890	8.260890+ $\frac{3}{1.0444}$ =11.133298	11.133298+ $\frac{1}{1.09142}$ =12.049536	X	X
2		X	X	X	X	X
3			8.260890 +$\frac{0.25+3}{1.13011}$ =11.136716	11.136716+ $\frac{1}{1.18096}$ =11.983185	11.983185+ $\frac{1}{1.24592}$ =12.785805	12.785805+ $\frac{1}{1.29575}$ =13.556804
4				11.133298+ $\frac{1+0.3}{1.26097}$ = 12.164250	12.164250+ $\frac{1}{1.133032}$ =13.046838	13.046838+ $\frac{1}{1.38354}$ =13.769622
5					11.983185+ $\frac{1+0.3}{1.36886}$ =12.932880	12.932880 $+\frac{1}{1.42361}$ =13.635320
6						12.785805+ $\frac{0.2+1}{1.49205}$ =13.590066
$V(\ell(t-1),t)\cup\ell(t-1)$	/	{1}	{3} ∪ {1}	{3,4} ∪ {1}	{4,5} ∪ {3}	{4,5,6} ∪ {3}
$\ell(t)$	1	1	1	3	3	3
m(t)	1	1	3	4	5	6
F(t)	3.282051	8.260890	11.133298	11.983185	12.785805	13.556804

II. Algorithm Execution Example II

j \ t	1	2	3	4	5	6
1	0.2+6 .975 =6.358974	6.358974+ 10 1.00425 =16.316654	16.316654+ 6 1.04442 =22.061470	X	X	X
2		X	X	X	X	X
3			16.316654 +0.25+6 1.13011 =21.847089	21.847089 +2 1.18096 =23.540626	23.540626 +2 1.24592 =25.145866	25.145866 +2 1.29575 =26.689374
4				21.847089 +0.3+2 1.26097 =23.671082	23.671082+ 2 1.33032 =25.174479	25.174479+ 2 1.38354 =26.620046
5					23.540626 +2+0.3 1.36886 =25.220856	25.220886 +2 1.42361 =26.625735
6						25.145866+ 2+0.20 = 1.49205 26.620347
V(ℓ(t-1), t)∪ℓ(t-1)	/	1	{1} ∪ {3}	{4}∪{3}	{4,5} ∪ {3}	{4,5,6} ∪ {3}
(t)	1	1	3	3	3	6
m(t)	1	1	3	4	5	6
F(t)	6.358974	16.316654	21.847089	23.540626	25.145866	26.6620347

Discussion

Example I shows no evidence of planning horizon. The only times where $m(t) = \ell(t)$ is in $t=1,2$, and $\ell(t) = 1$. This tells only that period $t=0$ can be planned optimally, but $t=0$ is not in the problem (we defined the problem to short in period $t=1$). The firm in this example has two alternatives - to invest more in forecasting research and come up with data for periods after $t=6$, or to issue in $t=1$ based upon the information at hand, knowing that it may be not optimal. In the second case the firm knows that based on the available information for 6 periods, the last issue is in $t=3$ ($\ell(6) = 3$), hence in period $t = 1$ it will issue for demand in periods $t=1,2,$. To illustrate the uncertainty involved suppose that by obtaining data for $t=7$ and solving the algorithm it is found that $m(7) = \ell(7)=4$ hence periods 1 through 3 can be scheduled optimally (in such a case $t=7$ is a forecasting horizon and $t=3$ is a planning horizon) hence we have a three periods problem with $\ell(3) = 1$, namely in $t=1$ issue for the demands in $t=1,2,3$. It is also possible that this planning hroizon will not be found by forecasting through $t=7$ but by information through $t=10$ (say). It is fairly possible that forecasting horizon will be found at $t=10$ such that $m(10) = \ell(10) = 9$ and $\ell(8) = 1$, namely, by obtaining additional data for $t=7,8,9,10$. We may find that it would be optimal to issue in $t=1$ for demand in $t=1,2,\ldots 8$. The firm will have to make a choice, in any case it will incur costs (costs related to a non-optimal solution or forecasting costs).

Consider now two firms identical to the firm in example I. No physical synergy exists between the two. By merging the two firms, the scheduling problem to be solved is the same in all the data except for the demand in each period are doubled. This is our example II. As we can see from the table II, in $t=3, \ell(3) = m(3) =3$, hence $t=3$ is a forecasting horizon and

$t=2$ is a planning horizon. Issuing in $t=1$ for demands in $t=1,2$ is optimal without further knowledge about $t=4,5,6$... furthermore, the merged firm can expect to have a second planning horizon (note that $m(6) = \ell(6)$) namely, to be able to make an optimal issue in $t=3$ for demand in $t=3,4,5$ based on data for periods 1 through 6.

By merging the two firms in our example all the uncertainty about the optimal issue in $t=1$ disappeared by forecasting up to $t=3$, and the uncertainty about optimal issue in $t=3$ was resolved by forecasting up to $t=6$.

For the following two propositions we skip the formal proofs.

Proposition 1

a. If firm A with Forecasting Horizon t_A and firm B with Forecasting Horizon t_B merge, the resulting Forecasting Horizon t_{AB} satisfies:

$$t_{AB} \leq \min(t_A, t_B)$$

b. The present value of forecasting costs after the merger is less then the present value of: the sum of forecasting costs for A and B minus the cost of forecasting those variables in A that have to be forecasted for B too.

Part b follows immediately from a. Part a holds because the only change due to the merger is an increase in the periodic demands. This will cause an increase in the tax bill and will motivate decrease in inventories by issuing for less periods ahead, hence the forecasting horizon cannot increase beyond $\min(t_A, t_B)$.

Clearly when merger takes place issuing costs will be lower due to some sharing of the fixed issuing costs by combining two separate issues into one. However, there is an additional effect - the merged firm may have a different optimal schedule than the sum of its parts (this is not shown in our example). Whenever this occurs, the savings are higher than the fixed cost saving,

(otherwise a different schedule will not occur). By the same reasoning that was used for Proposition 1, we have:

Proposition 2

The present value of equity issued by a merged firm is lower than: the sum of present values of equity issued separately, minus the present value of the fixed issuing costs in periods of parallel issuing.□

The above propositions provide a nonintuitive rationale for conglomerate merger. Although the magnitude of savings may be negligable, there are many cases where they are not. One such example is merger between small high gorwth companies or acquisition of a group of such companies are usually characterized by opportunities in their embryonic stages where forecasting of cash demands is difficult and costly and issuing costs that are high relative to their market values. Pooling together such firms increases the periodic cash demands and produces short planning horizons very quickly. The same effect will take place when a large and mature firm buys such growth companies.

Note also that the expected savings do not have to be distributed equally between the merging firms. An extreme example will be a case of one firm not gaining any benefit from the merger (the expected cost of obtaining its cash requirement is the same before and after the merger) while the other party gains all the benefits. The assymetry in the benefit sharing will be corrected by a suitable bonus from the party who gains most from the merger. If the acquiring party gains most we will witness an acquistion at premium, if the acquired party gains most the acquisition will be at discount, certeris puribus.

The Stochastic Case

The previous model was simplified to a great extent by assuming certainty. Once we remove these assumptions we must change the problem formulation and face major analytical difficulties. Unfortunately I could not resolve them. However, it is possible to justify the relevance of the previous model for real world problems, at least from a practical point of view. My purpose in this section is to point out the analytical difficulties of the stochastic problem, raise some questions for further research and finally suggest how to use the previous model in practical situations.

It is worthwhile to review the main assumptions of the previous model:

(1) Liquid assets were invested to yield a certain rate of return

(2) Cash demands were known with certainty

(3) The term structure was known with certainty

(4) Transaction costs were known with certainty

(5) Cash demands had to be satisfied on time.

The last assumption implies that the firm acts as if the penalty for not meeting demands on time is prohibitively high. The same constraint applied to a problem with stochastic demands implies setting the demands to their maximum level. This is highly unrealistic. The firm can always issue on a short notice, probably by paying higher transaction costs or satisfy the demand in a later period and suffer meanwhile a penalty due to delays in projects.

Assumption (1) seems to describe prudent behavior but is not necessarily optimal. The proper portfolio of liquid assets should be part of the optimal solution. It will probably not be a simple combination of the market portfolio and risk free assets. Unlike the assumptions of the C.A.P.M. the whole joint probability distributions of the portfolio return and the demands is relevant here as well as the cost structure of the problem. Intuition suggests that an optimal portfolio wll have high correlation with the demands, and will not be the same in each period.

The solution to this problem in a multi-period context seems to be extremely difficult to derive. However, it is an interesting problem for further research and may have interesting applications to more general problems.

In order to focus on the other major difficulty, I will model the stochastic problem under the assumption that the optimal liquid portfolio is known and yields a random return $\tilde{\gamma}_{p,t}$ after tax and the probability distribution of $\tilde{\gamma}_{p,t}$ is known in each period t. Demand is a random variable $\tilde{d}_t$ with known probability distribution in each t, and independent of $\tilde{\gamma}_{p,t}$. All the other parameters are known with certainty. There is a penalty P_t in period t per \$1 of unsatisfied cash demand, and demands can be satisfied in the following period.

The dynamic programming backwards algorithm can be written as:

1. $$J_N(I_N) = \frac{I_N}{\gamma_{1,N}}$$

2. $$J_t(I_t) = \min_{y_t \geq 0} \{\frac{\delta_t K_t + Y_t}{\gamma_{2,t}(1 - \alpha_t)} + \frac{1}{\gamma_{3,t}} E\{\max(0;\tilde{d}_t - Y_t - I_t)P_t$$

$$+ J_{t+1}[(I_t + Y_t - \tilde{d}_t)(1 + \tilde{\gamma}_{p,t})]\}\}$$

$$\text{s.t.} \quad \tilde{I}_{t+1} = (\tilde{I}_t + \tilde{Y}_t - d_t)(1+\gamma_{p,t})$$

$$I_o \text{ given}$$

where $J_t(I_t)$ is the cost to go (the optimal solution is given by $J_0(I_0)$), and $\gamma_{i,t}$ are the risk adjusted discount factors.

It is possible to show that the optimal issuing policy is of the (s_t, S_t) type,[2] namely, in each period t, issue if $I_t \geq s_t$:

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[2] For treatment of similar problem in the context of production see Chapter 3 in [1].

$$Y_t = S_t - I_t \quad \text{if } I_t < s_t$$

$$Y_t = 0 \qquad\quad \text{if } I_t \geq s_t$$

however, the analytical determination of s_t and S_t is difficult since they depend on the discount factors $\gamma_{i,t}$ which are hard to define (and among other things, depend on the optimal solution).

Clearly, relaxing the other simplifying assumptions will complicate the problem to such a level that in view of the inaccuracies in the data will make any attempt to optimally solve it, unrealistic.

Back to the Basic Model

In view of the above difficulties, the basic model may look attractive for solving practical problems. Using an exogenousely defined discount factor although theoretically wrong, is believed to be a reasonable procedure. We further recommend adjusting the discount factors to the risk of the demands. It is hard to quantify the qaulity of these approximate solutions. Such a quantification requires comparison with the optimal solution or at least to a superior numerical method in a variety of solution or at least to a superior numerical method in a variety of situations. However, some qualitative reasoning can support the assumption that it is a reasonable technique. First, note that the algorithm is extremely simple and calculations can be performed on calculator very fast. Sécond, the algorithm enables the identification of the forecasting and planning horizon. Once a forecasting horizon is identified, the solution for the periods defined by the planning horizon is not sensitive to introduction of uncertainties in periods after the forecasting horizon (hence the quality of the approximation is higher when a short forecasting horizon exist).

Third, although the identification of the forecasting and planning horizons assumes certainty in the data for the periods included in the forecasting horizon, in many cases there will be a wide range or variations in

the data for which the planning horizon will not change. Since it is so easy to execute the algorithm a "quick and dirty" sensitivity analysis is virtually costless.

It is worth repeating that since the model assumed certainty, the optimal solution will involve issuing only in periods with zero inventory. From the discussion on the nature of the solution to the stochastic case, we know that optimal solution is of the type (s_t, S_t) hence, any solution should involve some buffer inventory.[3] When using the deterministic algorithm we will not specify a buffer stock for each period but rather establish it once for the whole planning horizon. By performing the sensitivity analysis one can get pretty good intuition for the proper size of this inventory. This inventory is external to the algorithm. The algorithm will be performed as if no buffer inventory exist.

Summary

This article treated the effects of transaction costs on scheduling stock issues. Debt levels were pre-determined. However, Lemma 1 could be modified to hold for debt issues or a combination of debt and equity issues, leading to somewhat similar problem formulation.[4] Solving such a problem will provide a better understanding of the changing pattern of D/E ratios. Based on the planning horizon theorem it was possible to suggest that managers can solve the scheduling without having to do a lot of forecasting. A non-intuitive rationale for conglomerate merger was demonstrated, and economic conditions affecting the likelihood of existance of planning horizons were discussed. Using the deterministic model as a benchmark, an

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[3]This is clearly supported empirically. We don't see corporations issuing when they have no cash at all.

[4]One clear result for a problem that includes debt issues will be that the firm will borrow and lend at the same time in order to save transaction costs.

intuitively and practically appealing technique for solving the stochastic problem was suggested. However, major questions remain open, for example - what is the optimal portfolio of liquid assets the firm should hold? Unlike the individual investor whose objective is to hold a portfolio that maximizes its utility from consumption over time, the firms is motivated to hold a portfolio that will minimize the value of liabilities. The character of the cash demands and costs in each firm may suggest a different portfolio for each firm in each period.

Finally, the method for obtaining the planning horizon theorem can be implemented for more general dynamic lot size problem with appreciating or deteriorating inventories. This may be of interest to people in the area of production planning.

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